[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 2078 **GC-3** **Your Roll No................**

Unique Paper Code : 32371303

Name of the Paper : Mathematical Analysis

Name of the Course : **B.Sc. (H) STATISTICS (CBCS)**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. Attempt in all **4** Questions from **Section I** and **3** Questions from **Section II.**
3. Question numbers **1** and **6** are compulsory.
4. Use separate answer books for **Section I** and **Section II.**

Section – I

1. (a) Write the Supremum and Infimum of the following :

$$S = \left\{1 - \frac{1}{3^n}, n \in N\right\}$$

(b) (i) Give an example of a set (other than R and φ) which is a neighbourhood of each of its points.

(ii) Give an example of a closed set which is not an interval.

(c) State whether the following statements are True or False

(i) A finite set has no limit point.

(ii) A sequence can converge to more than one limit.

(d) Assuming that $n^{1/n} \to 1$ as $n \to \infty$. Show by applying Cauchy's n^{th} root test that the series $\sum_{n=1}^{\infty} \left(n^{1/n} - 1\right)^n$ converges.

(e) Give the geometrical interpretation of Lagrange's Mean Value Theorem.

(2×5)

2. (a) Show that the set of rational numbers is not order complete.

(b) Define neighbourhood of a point. If M and N are neighbourhoods of a point p, then prove that $M \cap N$ is also a neighbourhood of p. (6,6)

3. (a) Prove that $\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n$ exists and lies between 2 and 3.

(b) Show that every Cauchy sequence is bounded. (6,6)

4. (a) Test the series $1+\frac{x^2}{2}+\frac{x^4}{4}+\frac{x^6}{6}+\cdots$ for convergence for all positive values of x.

(b) Let f be the function defined on [0,1] by setting

$$f(x) = \frac{1}{2^n}, \text{ when } \frac{1}{2^{n+1}} \le x \le \frac{1}{2^n}, \quad n = 0,1,2,3,....$$

$$f(0) = 0$$

Examine the continuity of the function at the points $x = \frac{1}{2}, \frac{1}{2^2}, \cdots, \frac{1}{2^n}, 1$.

(6,6)

5. (a) Show that there is no real number k for which the equation $x^3 - 3x + k = 0$ has two distinct roots in [0, 1].

(b) Obtain Maclaurin's series expansion of e^x. (6,6)

Section – II

6. (a) If u_x be a function whose differences when the increment of x is unity are denoted by $\delta u_x, \delta^2 u_x, \ldots \ldots$ and $\Delta u_x, \Delta^2 u_x, \ldots \ldots$ when the increment of x is n, then what is the relation between δ and Δ.

(b) What is the value of $\Delta^4 f(x)$?, where $f(x) = x^3 + 4x^2 + 3$.

(c) Represent $f(x) = x^3 + 4x^2$ in terms of factorial notations.

(d) State properties of Cote's numbers.

(e) $\frac{1}{2}\delta^2 + \delta\sqrt{\left(1+\frac{1}{4}\delta^2\right)} \equiv \cdots$, where δ is a central difference operator. (1×5)

7. (a) Show that

$$y_4 = y_0 + 4\Delta y_0 + 6\Delta^2 y_{-1} + 10\Delta^3 y_{-1},$$ as far as third differences.

(b) Find the relation between α, β, γ in order that $\alpha + \beta x + \gamma x^2$ may be expressible in one term in the factorial notation.

(c) Show that

$$u_{2n} - \binom{n}{1}2^1 u_{2n-1} + \binom{n}{2}2^2 u_{2n-2} \cdots\cdots\cdots + (-2)^n u_n = (-1)^n (c - 2an),$$

where $u_x = ax^2 + bx + c$. (4×3)

8. (a) Obtain Lagrange's interpolation formula in the form

$$f(x) = \sum_{r=1}^{n} \frac{L(x).f(x_r)}{(x - x_r).L'(x_r)} = \sum_{r=1}^{n} L_r(x).f(x_r),$$

where $L(x) = (x - x_1)(x - x_2) \ldots (x - x_n)$.

(b) Prove that

$$\Delta^r f_i = \delta^r f_{i+\frac{r}{2}} = \nabla^r f_{i+r} = r!h^r f[x_i, x_{i+1} \ldots\ldots\ldots\ldots x_{i+r}],$$

where $f_i = f(x_i)$. (6,6)

9. (a) Prove that, if f(x) is a continuous function whose fifth differences are constant,

then $\int_{-1}^{1} f(x)dx = \frac{8}{9}f(0) + \frac{5}{9}\left\{f\left(\sqrt{0.6}\right)+f\left(-\sqrt{0.6}\right)\right\}$.

(b) Solve any **two** of the following difference equations :

(i) $u_{x+2} + a^2u_x = \cos ax$

(ii) $u_{x+2} - 7u_{x+1} + 10u_x = 12.4^x$

(iii) $u_{x+4} + u_x = 0$ (6,6)